String Gas Cosmology and Non-Gaussianities

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Abstract

Recently it has been shown that string gas cosmology, an alternative model of the very early universe which does not involve a period of cosmological inflation, can give rise to an almost scale invariant spectrum of metric perturbations. Here we calculate the non-Gaussianities of the spectrum of cosmological fluctuations in string gas cosmology, and find that these non-Gaussianities depend linearly on the wave number and that their amplitude depends sensitively on the string scale. If the string scale is at the TeV scale, string gas cosmology could lead to observable non-Gaussianities, if it is close to the Planck scale, then the non-Gaussianities on current cosmological scales are negligible.

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1 Introduction

One of various alternative scenarios to inflation is string gas cosmology (SGC) [1, 2]. Unlike the usual inflationary models, SGC does not assume a quasi-de Sitter phase in which the universe undergoes an exponential growth. Instead, in SGC, there exists a quasi-static Hagedorn phase, during which thermal fluctuations of closed strings generate the density perturbations which seed the wrinkles in the cosmic microwave background (CMB) and the large scale structure of our universe today. In [3, 4], it was shown that metric perturbations in SGC yield a scalar power spectrum consistent with current experiments, and predict a slight blue tilt to the spectrum of gravitational waves [5].

SGC can be embedded naturally into string theory, which is the most promising candidate for quantum gravity. Potentially, SGC can overcome some of the conceptual problems of the inflationary scenario such as the singularity problem [6] and the trans-Planckian problem for fluctuations [7].

SGC is based on taking into account the new degrees freedom (oscillatory and winding modes) and new symmetries (T-duality) of string theory. The oscillatory modes lead to a maximal temperature (Hagedorn temperature [8]) which a gas of strings can attain. In turn, the presence of a maximal temperature gives rise to the hope that in SGC one may avoid the cosmic singularity. Also, through the dynamical process of collision of string winding modes, our universe can naturally evolve from (9+1) or (10+1) compact dimensions to a space-time with (3+1) large dimensions where we live today [1, 2]. In short, SGC opens a new window to study the early universe. Various issues have been discussed in the literature (see [9, 10, 11] for reviews and references to other work on string gas cosmology). In particular, string gas cosmology provides an elegant mechanism to stabilize most of the string moduli [14, 15, 16, 17, 18, 19], the one exception being the dilaton field. The dynamics of the compact dimensions relative to our three large spatial ones is studied in more detail in [20, 21, 22].

In this paper, we would like to discuss the non-Gaussianities in string gas cosmology. Non-Gaussianity is one of the most important quantities which can be measured in upcoming experiments. Non-Gaussianity is characterized by amplitude, shape, sign

and even running. Thus, it may contain a lot of information about the very early universe, and can be used to rule out models and set constraints on model building.

In practise, the amount of non-Gaussianity is often described using the quantity $f_{\rm NL}$ [23], which is of the order

$$f_{\rm NL} \sim \frac{\mathcal{R}_g - \mathcal{R}}{\mathcal{R}_g^2 - \langle \mathcal{R}_g^2 \rangle} , \tag{1}$$

where $\mathcal{R}$ is the comoving curvature perturbation, and the subscript g denotes its Gaussian part.

In [24], we developed a method to calculate the non-Gaussianities of the fluctuations with thermal origin. Since in string gas cosmology matter in the early Hagedorn phase is dominated by a thermal gas of strings, the perturbations are of thermal origin. Hence, the techniques of [24] can be used to study the non-Gaussianities in SGC and see under which conditions one obtains a value of f_{NL} which is in the range of anticipated observations.

This paper is organized as follows. In Section 2, we describe the evolution of the space-time background during the Hagedorn and radiation-dominated phases. In Section 3, we use thermal correlation functions to calculate the power spectrum of scalar metric perturbation and the non-Gaussianities.

2 Space-time Background in String Gas Cosmology

In string gas cosmology, the radiation phase of standard cosmology is preceded by the Hagedorn phase, a phase during which the temperature hovers near its maximal value, the Hagedorn temperature. Based on the symmetries of string theory, it is reasonable to assume that this phase is quasi-static in the sense that both the scale factor and the dilaton are constant. Einstein gravity is inconsistent with this behavior since it does not contain one of the key symmetries of string theory, T-duality. A better action to use is dilaton gravity, given by the effective action

$$S = - \int d^{d+1}x \sqrt{-g} e^{-2\phi} [\Re + 4 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi] , \tag{2}$$

where $\Re$ is the Ricci scalar in the string frame, g is the determinant of the metric of the $d+1$ dimension space-time, and ϕ is the dilaton field. The topology of space is $T^d \times T^{9-d}$, where $d = 3$ is the number of the large spatial dimensions, and the other $9 - d$ spatial dimensions are stabilized at a microscopic scale. The above effective action only describes the expanding space-time. The conditions to use the above action are that the string coupling is small and the moduli of the extra compact small dimensions are stabilized. Similar to what is done in Friedmann cosmology, the early universe is assumed to be homogeneous and isotropic, given by the metric

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \, . \tag{3}$$

Through direct calculation, it follows that the variational equations of motion in the string frame can be expressed as [2],

$$\begin{aligned} -(d)\dot{\lambda}^2 + \dot{\varphi}^2 &= e^{\varphi} E & (4) \\ \ddot{\lambda} - \dot{\varphi}\dot{\lambda} &= \frac{1}{2} e^{\varphi} P & (5) \\ \ddot{\varphi} - (d)\dot{\lambda}^2 &= \frac{1}{2} e^{\varphi} E \, , & (6) \end{aligned}$$

where E and P are the total energy and pressure in $(d+1)$ dimension space-time, and for convenience we introduced $\lambda = \ln[a(t)]$ as the logarithm of the scale factor, and rescaled the dilaton $\varphi = 2\phi - d\lambda$.

The dynamics of space-time in SGC results by coupling a thermal gas of strings as matter (determining the energy and the pressure) to the above background. The new degrees of freedom and new symmetry of string theory lead to a cosmology which is rather different from what is obtained in standard cosmology. Let us follow the universe back in time. At large radii, all of the energy of the strings is in string modes which behave as usual radiation. It can be shown that in the case of a radiative equation of state the above background equations reduce to the usual Einstein equations, and the dilaton can be set to be constant. Thus, at late times SGC and Standard Big Bang cosmology predict the same evolution of the scale factor. However, at small radii the string energy will begin to flow into the winding modes. This, in turn, will lead to a reduction in the pressure. When the radius equals the string scale (or, more generally, when the energy density exceeds the string energy density), there is an equal amount of energy in the winding and momentum modes, and their contribution

to the pressure cancels. It the total pressure vanishes, then, as follows immediately from (5), the scale factor tends to a constant. Thus, at high densities the universe hovers in a quasi-static Hagedorn phase.

One set of relevant new degrees of freedom are the winding modes of the closed string, and the new symmetry is the T-duality. To illustrate these new features, let us consider string theory on a circle T^1 with radius R. On this background, there are three classes of string modes: oscillatory modes, momentum modes and winding modes. The momentum modes correspond to the momentum of the center of mass of the string, and is analogous to the corresponding momentum states for a point particle on T^1. The energies of the momentum modes take on the values $\frac{n}{R}$ (with positive integers n) due to usual quantization of momenta on this compact background. The winding modes correspond to the number of times the closed string winds around T^1. Their energies are quantized as mR, where the positive integer m is the number of times the string winds the T^1. Since the string oscillatory modes have energies which are independent of R, the spectrum of the perturbative string is invariant under so-called T-duality:

$$R \to \frac{1}{R}, \;\; (n,m) \leftrightarrow (m,n)\,. \tag{7}$$

The vertex operators also obey this symmetry, and by introducing branes T-duality can be extended to be a symmetry of the full string theory: it is not just a perturbative symmetry, but also a non-perturbative one (see e.g. [25, 26]).

As the temperature of the string gas increases, the number of accessible string oscillatory modes increase exponentially. This leads to the existence of a maximal temperature for a gas of strings in thermal equilibrium, the Hagedorn temperature T_H [8]. This limiting temperature is reached once the energy density reaches string density. Because of T-duality, the temperature of a string gas is maximal if R is equal to the string length l_s. Moreover, the higher the entropy of the string gas is at the duality point $R = l_s$, the larger the region of R values for which the temperature hovers near the Hagedorn temperature.

The phase during which the temperature is close to the Hagedorn temperature and the string gas contains a thermal distribution of both winding and momentum modes is called the Hagedorn phase. During this phase, the winding modes and the

momentum modes contribute with opposite sign to the pressure, so that the equation of state is $\omega = 0$ (modulo the contribution of the oscillatory modes). Considering the above fact, it follows from the equations of motion (4-6) that the string frame scale factor has a static solution. The evolution of the dilaton is given by

$$\ddot{\phi} + (d)\dot{\lambda}\dot{\phi} - 2\dot{\phi}^2 \ = \ -\frac{1}{2}e^{\varphi}E[1-(d)w] \ . \tag{8}$$

In the Hagedorn phase, the dilaton cannot be static, unless the effective field theory for the background fields is changed. In the radiation phase, on the other hand, the equations of motion can be combined to show that the dilaton approaches a constant due to the Hubble damping term.

As discussed in [27] (see also [28]), the time dependence of the dilaton in the Hagedorn phase which follows from the equations of motion for dilaton gravity conflicts with the basic symmetries on which string cosmology must be based. Once the dilaton becomes large, the string gas should include Dp branes which become light. The S-duality symmetry of string theory indicates that one should expect that the dilaton is static in the Hagedorn phase [27]. In order to stabilize the dilaton, extra ingredients are needed. One attempt to find a background action which is consistent with the idea of a Hagedorn phase in which both the scale factor and the dilaton are quasi-static was recently made in [29]. It was based on ideas of tachyon condensation, i.e. introducing a moving tachyon condensate. In the context of higher derivative gravity actions which admit a singular bounce [30], it is possible to obtain a Hagedorn string gas phase near the bounce point in a model without a dilaton [31]. It is still an open issue how to stabilize the dilaton more naturally.

The transition between the Hagedorn phase and the radiation phase of standard cosmology is smooth and is triggered by the annihilation of winding and anti-winding modes into string loops (which behave like radiation). Whereas in the Hagedorn phase the scale factor is almost constant, in the radiation phase it evolves as

$$a(t) \ \sim \ t^{\frac{1}{2}} \ . \tag{9}$$

Therefore, the Hubble radius is almost infinite in the Hagedorn phase as shown in Fig.1. During the transition between the Hagedorn phase and the radiation phase the Hubble radius rapidly decreases. Once in the radiation phase, the Hubble radius

increases linearly in time as in standard cosmology. Because of the static nature of the Hagedorn phase, all scales of cosmological interest today are sub-Hubble during this period. Provided that the Hagedorn phase is sufficiently long, thermal equilibrium can be established on these scales.

Comoving scales whose physical length today corresponds to the current Hubble radius had a physical wavelength of the order of 1mm at the end of the Hagedorn phase, assuming that the temperature at that time was of the order of the scale of Grand Unification. The length is constant during the Hagedorn phase since the universe is static. The initial conditions for fluctuations in SGC are very different than in inflationary cosmology. In inflationary cosmology, the exponential expansion of space red-shifts any pre-existing matter, leaving matter in a local vacuum state. Thus, fluctuations are quantum vacuum type. In contrast, in SGC matter is not red-shifted during the Hagedorn phase. If matter is in a thermal state, then the fluctuations will be string thermodynamic fluctuations. Since we are interested in length scales which are in the far infrared compared to the string scale, it is reasonable to assume that the fluctuations can be treated using Einstein gravity. Given this assumption, it was discovered in [3] that the spectrum of resulting cosmological perturbations today is roughly scale-invariant with a small red tilt. The spectrum of gravitational waves is also almost scale-invariant, but has a slight blue tilt [5], yielding a distinctive signature of the SGC structure formation scenario for experiments. Since the fluctuations exit the Hubble radius and evolve outside of the Hubble radius as they do in inflationary cosmology, they are squeezed and yield acoustic oscillations in the angular power spectrum of cosmic microwave background anisotropies, again as in inflationary cosmology (see e.g. [32] for a comprehensive survey of the theory of cosmological perturbations and [33] for an overview).

3 Thermal Correlation Functions and Non-Gaussianity

In this section, we calculate the two- and three-point correlation functions of the scalar metric perturbations, and discuss implications for future experiments.

At the end of the Hagedorn phase, the Hubble scale is decreasing from near infinity to a very small scale. After the winding modes annihilate with each other and create

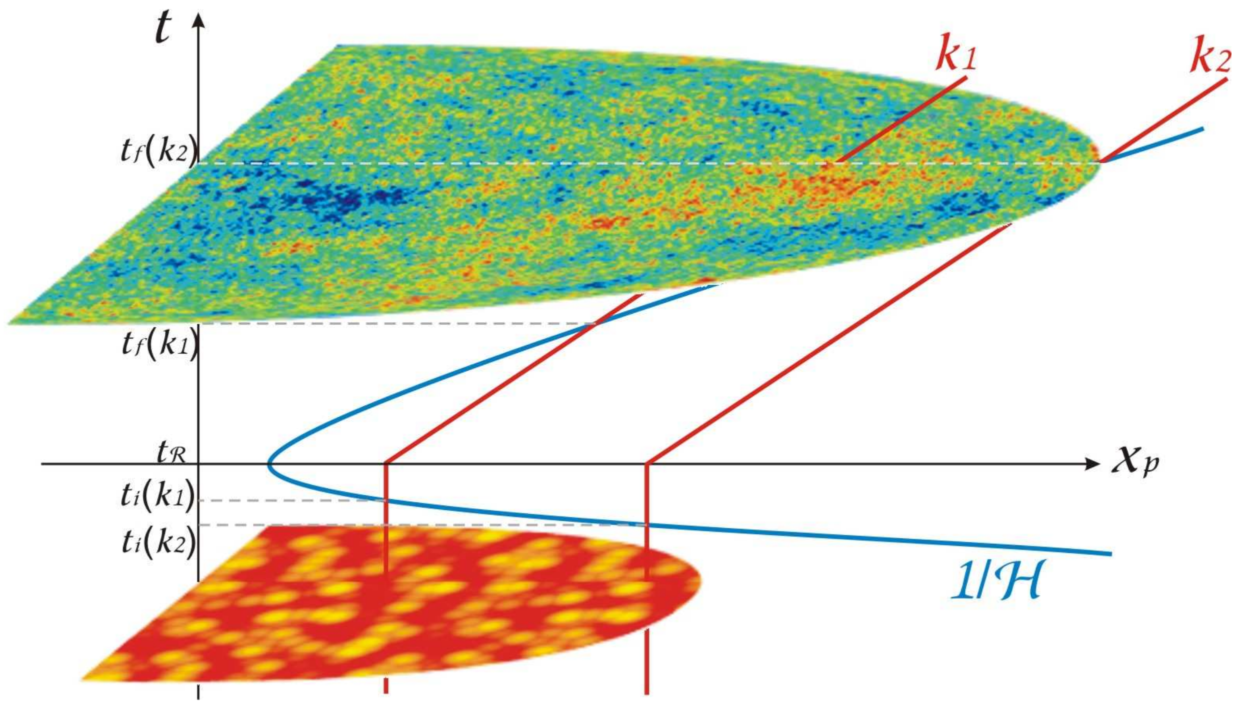

Figure 1: The figure illustrates how the Hubble radius evolves from the Hagedorn phase to the radiation-dominated period. Different modes in the Hagedorn phase escape the Hubble radius with different physical length, and they re-enter the Hubble radius to generate anisotropies in the CMB.

more and more radiation, the universe naturally evolves into the radiation dominated phase. Since during the Hagedorn phase the Hubble radius is much larger than the physical length of scales we are interested in, we are in the sub-Hubble regime when matter fluctuations dominate the dynamics. The logic of the calculation is to compute the matter perturbations during the Hagedorn phase, convert them to metric fluctuations once the scales exit the Hubble radius at the end of the Hagedorn phase, and then to evolve the metric fluctuations as is usually done in cosmological perturbation theory. This is the logic espoused in [3, 4]. We will thus begin with the computation of the matter fluctuations.

Since the Hagedorn phase is quasi-static and adiabatic, and matter is dominated by the thermal gas of strings, one can analyze the thermodynamics of these strings and calculate the thermal fluctuation in the standard way. Starting point for the computation of the thermal fluctuations is the expression for the partition function of a gas of closed strings on a compact space with three large spatial dimensions derived in [34] (see also [35, 36] for other work on string gas thermodynamics). The three large spatial dimensions are assumed to have toroidal topology such that stable string winding modes exist.

The correlation functions of the density fluctuation can be obtained from the partition function Z

$$Z \;=\; \Sigma \exp(-\beta E_\alpha) \ , \tag{10}$$

where $\beta = T^{-1}$. The first step of the analysis is to compute the average energy of a gas of closed string in a box of length L, which in the limit when the temperature T is very close to the Hagedorn temperature is given by [37, 3, 4]

$$\langle U \rangle \;\simeq\; \frac{L^2}{l_s^3} \ln\left[\frac{l_s^3 T}{L^2(1-\frac{T}{T_H})}\right] \ , \tag{11}$$

where T_H is the Hagedorn temperature.

Then, the two-point correlation function for the fluctuation

$$\delta\rho \;\equiv\; \rho - \langle\rho\rangle \tag{12}$$

is given by [3, 37, 4]

$$\langle \delta\rho^2 \rangle \;=\; \frac{\langle \delta U^2 \rangle}{V^2} \;=\; \frac{1}{V^2}\frac{d^2 \log Z}{d\beta^2} \;=\; -\frac{1}{V^2}\frac{d\langle U \rangle}{d\beta} \;=\; \frac{T}{L^4 l_s^3 (1-\frac{T}{T_H})} \ . \tag{13}$$

Similarly, the three-point correlation function can be expressed as [24]

$$\langle \delta\rho^3 \rangle \;=\; \frac{\langle \delta U^3 \rangle}{V^3} \;=\; -\frac{1}{V^3}\frac{d^3 \log Z}{d\beta^3} \;=\; \frac{1}{V^3}\frac{d^2\langle U \rangle}{d\beta^2} \;=\; \frac{T^2}{L^7 l_s^3 (1-\frac{T}{T_H})^2} \; . \tag{14}$$

Note that the linear density fluctuations is highly gauge-dependent on super-Hubble scales. Here, however, we are only following the matter perturbations on scales smaller than the Hubble radius. On these scales, the gauge-dependence is negligible.

In the transition period between the Hagedorn phase and the radiation phase, the Hubble radius decreases fast, but the physical length of the fluctuations is almost unchanging. Once the fluctuations leave the decreasing Hubble radius, the fluctuations freeze out. During the radiation phase, the perturbations red-shift as standing waves, but maintain the information about their thermal origin in the initial Hagedorn phase. Since the fluctuations re-enter the Hubble radius at late times as coherent standing waves, they will induce acoustic oscillations in the angular power spectrum of CMB anisotropies in the same way as happens in inflationary cosmology.

The density fluctuations $\delta\rho_{\mathbf{k}}$ in momentum space are given by those in position space via

$$\delta\rho_{\mathbf{k}} \;=\; k^{-\frac{3}{2}}\delta\rho \; . \tag{15}$$

When scales exit the Hubble radius, we determine the induced metric perturbation using the Einstein constraint equations (we are making the assumption that the fluctuations on the infrared scales relevant to current observations are indeed described by the Einstein equations). As shown in detail in [4] the scalar metric fluctuation Φ is determined via

$$\Phi_{\mathbf{k}} \;\sim\; 4\pi G \delta\rho_{\mathbf{k}} (\frac{a}{k})^2 \; , \tag{16}$$

where G is Newton's constant, and $\Phi_{\mathbf{k}}$ is the Fourier mode of the longitudinal gauge metric perturbation defined via the following expression for the metric including scalar metric inhomogeneities (in the absence of anisotropic stress):

$$ds^2 \;=\; a^2 \left(-(1+2\Phi)d\eta^2 + (1-2\Phi)d\mathbf{x}^2 \right) \; . \tag{17}$$

Note that in the gauge we are using, on super-Hubble scales the fluctuations in Φ and $\delta\rho$ have the same spectrum, the factor k^2 in (16) being replaced by the square of the Hubble paramter $\mathcal{H}$ in conformal time. However, at Hubble radius crossing $\mathcal{H} = k$, and thus we obtain the Poisson-like equation (16).

Since the equation of state parameter $1 + w$ (where w is the ratio of pressure to energy density) does not change substantially, the amplitude of the metric perturbation is almost constant after it leaves the Hubble radius. Thus, we can calculate the late-time power spectrum of cosmological fluctuations from the Hagedorn phase matter perturbations making use of the constraint equation (16) and the Hubble radius crossing condition $k/a(t_H(k)) = H(t_H(k))^{-1}$, yielding

$$\langle \Phi_{\mathbf{k}}^2 \rangle \sim (4\pi G)^2 \frac{T}{l_s^3(1 - \frac{T}{T_H})} k^{-3} , \tag{18}$$

where $t_H(k)$ is the time when the scale k exits the Hubble radius during the transformation period.

Therefore the power spectrum of $\Phi_{\mathbf{k}}$ can be expressed as

$$P_\Phi \equiv \frac{k^3}{2\pi^2} \langle \Phi_{\mathbf{k}}^2 \rangle \simeq 8G^2 \frac{T}{l_s^3(1 - \frac{T}{T_H})} \simeq 8(\frac{l_p}{l_s})^4 \frac{1}{(1 - \frac{T}{T_H})} , \tag{19}$$

where l_p is the Planck length, and the temperature T is to be evaluated at the time $t_H(k)$.

Similarly, the three-point correlation function of $\Phi_{\mathbf{k}}$ after leaving thermal equilibrium is expressed as

$$\langle \Phi_{\mathbf{k}}^3 \rangle \sim (4\pi G)^3 \frac{T^2 H(t_H(k))}{l_s^3(1 - \frac{T}{T_H})^2} k^{-9/2} . \tag{20}$$

When the physical wavelength of the mode k is much larger than the Hubble radius, the usual practise in cosmological perturbation theory is to focus on the conserved quantity $\mathcal{R}_{\mathbf{k}}$, the curvature fluctuation in co-moving gauge, given by

$$\mathcal{R} \simeq \zeta = \Phi + \frac{2}{3} \frac{H^{-1}\dot{\Phi} + \Phi}{1 + w} . \tag{21}$$

The constancy of $\mathcal{R}$ is then used to relate early time to late time fluctuations. Since in the Hagedorn phase $w = 0$ and in the radiation dominated universe $w = 1/3$, then, up to a order one constant, the constancy of $\mathcal{R}$ implies the constancy of Φ, the variable which determines the CMB anisotropies.

Thus, we can directly compute the non-Gaussianity estimator $f_{\rm NL}$, and it takes the form

$$f_{\rm NL} \sim k^{-\frac{3}{2}} \frac{\langle \mathcal{R}_{\mathbf{k}}^3 \rangle}{\langle \mathcal{R}_{\mathbf{k}}^2 \rangle \langle \mathcal{R}_{\mathbf{k}}^2 \rangle} \sim \frac{l_s^3 H(t_H(k))}{4\pi l_p^2} . \tag{22}$$

Using the condition of modes escaping and re-entering the Hubble radius $k = a(t_H(k))H(t_H(k)) = aH$ and $k_0 = a_0 H_0$, and considering the relationship between scale factor and temperature, we can estimate the value of the non-Gaussianity to be

$$f_{\rm NL} \simeq (\frac{l_s}{l_p})^2 \frac{H}{T} = (\frac{l_s}{l_p})^2 \frac{H_0}{T_0} \frac{k}{k_0} \simeq (\frac{l_s}{l_p})^2 \times 10^{-30} \frac{k}{k_0} \ , \tag{23}$$

where the subscript 0 on H,T, and k represent today's values, subscript t represents the values of Hagedorn phase.

From the above relation, we see that the non-Gaussianity estimator f_{NL} depends linearly on the mode k. This reflects the fact that thermal string fluctuations lead to non-Gaussianities which are of order unity on microscopic scales, but which are suppressed on larger scales in accordance to the Central Limit Theorem. This means that modes reentering the Hubble radius earlier have a larger non-Gaussianity. This is very different from what happens in inflationary models, where the non-Gaussianities are almost scale invariant.

Moreover, f_{NL} depends sensitively on the string scale . If the string scale is high (comparable to the Planck scale or the scale of Grand Unification), then f_{NL} will be be too small to be observed in the future experiments. However, if the string scale is at the TeV scale, then the non-Gaussianity on a scale of k_0 is $f_{\rm NL} \sim \mathcal{O}(1)$. For such a string scale, however, it requires fine tuning of the temperature T of the string gas in the Hagedorn phase to obtain a power spectrum with reasonable amplitude. Therefore if the future experiment obtains a non-vanishing non-Gaussianity, this would pose a significant challenge to string gas cosmology. On the other hand, if non-Gaussianity with an amplitude growing linearly with k were to be detected, it would provide a strong signature in support of string gas cosmology with a low string scale.

4 Conclusion and Discussion

In this paper, we have calculated the non-Gaussianity parameter f_{NL} in string gas cosmology using the formalism developed in [24]. We obtain a result which grows linearly with k. For a high string scale, the amplitude of the predicted non-Gaussianities, however, is much too small to be observable on cosmological scales today.

In general inflation models, the shape of the non-Gaussianity [23] is almost scale invariant, since during the inflation period different modes exit the Hubble radius

in almost the same physical environment (the time-translation invariance of the De Sitter phase). In contrast, in SGC the fluctuations exit the Hubble radius in a phase in which the string gas is rapidly changing its character. Although the holographic scaling of the specific heat of a closed string gas can ensure the scale-invariance of the power spectrum, the three-point correlation function cannot be scale-invariant.

If the string scale is high, SGC cannot give a large non-Gaussianity. The amplitude could be very hard if not completely impossible to detect in future experiments. If the string scale is decreased to the TeV scale, which could happen in some scenarios with warped geometry, then the non-Gaussianity could be large. However, in this one would require a fine-tuning on the temperature T of the thermal string gas in order to obtain a power spectrum with reasonable amplitude.

We should also emphasize that the method used here to calculate the three-point correlation function in SGC is general. It can be applied to inflationary scenarios with thermal fluctuations. The very early universe has many thermal elements, whose fluctuations could be the source of non-Gaussianities in the CMB. In the case of inflation, the physical wavelength of fluctuations when they exit the Hubble radius is microscopic, compared to the situation in SGC where the length is macroscopic. This leads to a much larger amplitude of the non-Gaussianities in thermal inflation models, as shown in [24].

We wish to mention a further caveat to our analysis: we have neglected the possible existence of cosmic superstrings [38] as a remnant of the early Hagedorn phase. Such strings would induce extra contributions to the spectrum of fluctuations and would lead to non-Gaussianities.

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